Short Papers

A Generalized Theory for Asymmetrical Coupled Lines

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Abstract **—A method will be presented that calculates the properties of geometrically or electromagnetically asymmetrical coupled lines. Analogous to the normally used theory of even and odd modes, the electromagnetic fields on the coupler will be described in terms of two modes as the basic variables to calculate the coupling behavior. As a criterion for determining the amplitudes of the two modes, the power transported on the two guides will be used. An image guide coupler with a ferrite slab will theoretically demonstrate the feasibility of this method.**

I. Introduction

Directional couplers are used in most millimeter-wave and microwave systems for comparing the power and phase of signals. Normally the couplers are symmetric but for some applications geometrically as well as electromagnetically asymmetric ones are needed. Thus, for example, the coupling properties can be influenced by the premagnetization of anisotropic ferrites. For such configurations the method presented here will extend the well-known theory of even and odd modes. A superposition of the fundamental and first higher order mode—normally these two modes are similar to even and odd modes on symmetrical structures—will be used to determine the transported power in the two guides. Thus incomplete coupling, i.e., a state where the power is never completely coupled to the other guide, can be calculated, too.

II. Theoretical Background

The conventional theory of even and odd modes [1] requires a symmetry plane in which electric and magnetic walls can be assumed. For some waveguides, such as finlines and slotlines, a strong field concentration exists within the slots [7]. The y components of the electric field strengths for both the even- and the odd-mode cases are symmetrical. It can be seen that a simple addition of these two modes at $z=0$ delivers a zero field on guide B which is equivalent to exciting the coupler structure at port 1 with all other ports matched. From this the well-known formulas for the scattering parameters are

$$S_{21}=\frac{U_2}{U_1}=\cos\left(\frac{\pi}{2}\frac{z}{L_0}\right)\exp\left(-j\frac{\pi}{2}\frac{z}{L_0}\right) \tag{1}$$

$$S_{31}=\frac{U_3}{U_1}=\sin\left(\frac{\pi}{2}\frac{z}{L_0}\right)\exp\left(-j\frac{\pi}{2}\frac{z}{L_0}-j\frac{\pi}{2}\right) \tag{2}$$

with the 0 dB coupling length

$$L_0=\frac{\pi}{\beta_{\text{even}}-\beta_{\text{odd}}}. \tag{3}$$

Manuscript received August 1, 1987; revised September 20, 1988.
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IEEE Log Number 8825387.

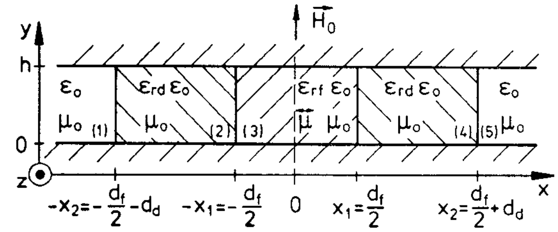

Fig 1. Cross section of an I guide coupler with a premagnetized ferrite slab

For asymmetrical coupled lines this theory cannot be used. One possibility for solving this problem is the application of the complete field matching technique. In this case higher order modes must be added to satisfy the field continuity conditions at all four ports [2]. This method is very time consuming but it is a rigorous solution of the problem. Another possibility is the description using characteristic impedances describing the first two modes [3]. This method can only be used for guides with well-defined characteristic impedances: i.e., current or voltage must be defined uniquely. But for transmission lines such as image guides with anisotropic media, the definition of the characteristic impedances is arbitrary. In these cases the modified theory of even and odd modes, which is presented here, is applicable.

The electromagnetic coupling between two asymmetrical image guides has been investigated in [5]. This structure not only has a large bandwidth in relation to symmetrical couplers; it also shows a very high directivity. New applications of this structure can be found by the additional use of premagnetized ferrite material within the structure, as developed in [5] with a longitudinally premagnetized ferrite. As an example for an asymmetrical coupler an I guide coupler with a transversally premagnetized ferrite will be investigated here (Fig. 1). The electric field of the fundamental mode (quasi-TEM) on this line has only a y component and is independent of the y coordinate [4], [6]. Such fields are excited by feeding the structure with a TE_{10} rectangular waveguide mode. The distributions of the y component of the electric field strength for mode I (similar to the even mode) and mode II (similar to the odd mode) are not symmetrical. It can be seen that the electric field does not completely vanish on one guide for any linear combination of these two modes. So an integral description, using the power transported into the z direction, must be used. First the power coupling of both modes has to be determined. The electromagnetic fields are a linear superposition of these two modes including an unknown real factor Q:

$$\vec{E}(x,y,z)=\vec{E}^{\text{I}}(x,y)\exp(-j\beta^{\text{I}}z)+Q\vec{E}^{\text{II}}(x,y)\exp(-j\beta^{\text{II}}z) \tag{4}$$

$$\vec{H}(x,y,z)=\vec{H}^{\text{I}}(x,y)\exp(-j\beta^{\text{I}}z)+Q\vec{H}^{\text{II}}(x,y)\exp(-j\beta^{\text{II}}z) \tag{5}$$

with $\vec{E}^{\text{I}}$, $\vec{E}^{\text{II}}$, $\vec{H}^{\text{I}}$, and $\vec{H}^{\text{II}}$ the real transversal structure functions of the electromagnetic fields of both modes and with β^{I}

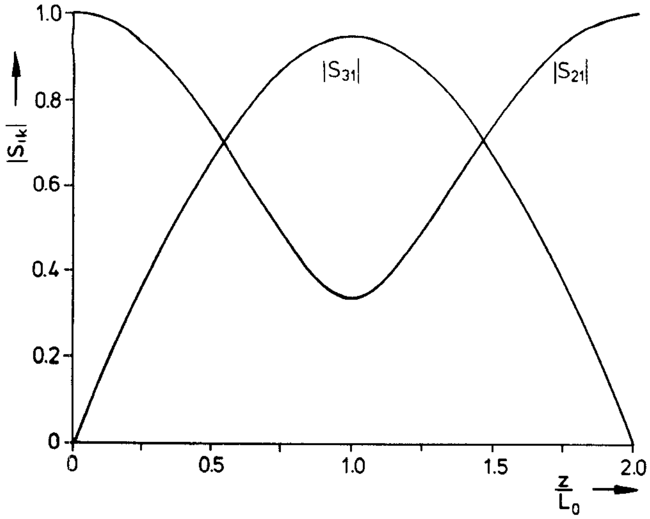

Fig. 2. Magnitudes of the scattering parameters versus the normalized coupling length z/L_0 for a power division factor $k = 0.8$.

and β^{II} the corresponding phase constants. Thus the power transported along each line is defined by

$$P_i = \iint_{A_i} 0.5\,\mathrm{Re}\{\vec{E}\times\vec{H}^*\}\cdot\vec{e}_z\,dA, \qquad i = A, B \tag{6}$$

with A_i the cross section of guide i. Solving the integrals, the transported power on guides A and B can be written as:

$$P_{\mathrm{A}}(z) = P_{\mathrm{A}}^{\mathrm{I,I}} + Q^2 P_{\mathrm{A}}^{\mathrm{II,II}} + Q P_{\mathrm{A}}^{\mathrm{I,II}}\cos(\pi z/L) \tag{7}$$

$$P_{\mathrm{B}}(z) = P_{\mathrm{B}}^{\mathrm{I,I}} + Q^2 P_{\mathrm{B}}^{\mathrm{II,II}} + Q P_{\mathrm{B}}^{\mathrm{I,II}}\cos(\pi z/L) \tag{8}$$

with the "0 dB coupling length"

$$L = \frac{\pi}{\beta^{\mathrm{I}} - \beta^{\mathrm{II}}} \tag{9}$$

corresponding to (3). The subscripts A and B indicate the guide, the superscripts I,I and II,II signify that these portions are caused by modes I and II, respectively. The superscript I,II means that this power is produced by the interaction of the two modes. These coefficients can be calculated by using the field distributions of the different modes, which again are known from the field theory which is used to determine the phase constants. Because the electromagnetic fields at $z = 0$ are assumed to have zero phase, the power constants are real values, too.

Because of power conservation the sum of both power portions must be independent of the z coordinate; the transmission lines are assumed to be lossless. Thus $P_{\mathrm{A}}^{\mathrm{I,II}}$ must be equal to $-P_{\mathrm{B}}^{\mathrm{I,II}}$:

$$P(z) = P_0 = P_{\mathrm{A}}(z) + P_{\mathrm{B}}(z) \Rightarrow P_{\mathrm{A}}^{\mathrm{I,II}} = -P_{\mathrm{B}}^{\mathrm{I,II}}. \tag{10}$$

This is always guaranteed because the two modes are orthogonal over the whole cross section $A = A_{\mathrm{A}} + A_{\mathrm{B}}$.

Now it will be assumed that the coupler is fed at port 1 and that all other ports are matched:

$$P_{\mathrm{A}}(z=0) = P_{\mathrm{A}}^{\mathrm{I,I}} + Q^2 P_{\mathrm{A}}^{\mathrm{II,II}} + Q P_{\mathrm{A}}^{\mathrm{I,II}} = P_0 \tag{11}$$

$$P_{\mathrm{B}}(z=0) = P_{\mathrm{B}}^{\mathrm{I,I}} + Q^2 P_{\mathrm{B}}^{\mathrm{II,II}} - Q P_{\mathrm{A}}^{\mathrm{I,II}} = 0. \tag{12}$$

Consequently, the unknown factor Q becomes

$$Q_{1,2} = \left(P_{\mathrm{A}}^{\mathrm{I,II}} \pm \sqrt{\{P_{\mathrm{A}}^{\mathrm{I,II}}\}^2 - 4P_{\mathrm{B}}^{\mathrm{I,I}}P_{\mathrm{B}}^{\mathrm{II,II}}}\right)\Big/\left(2P_{\mathrm{B}}^{\mathrm{II,II}}\right). \tag{13}$$

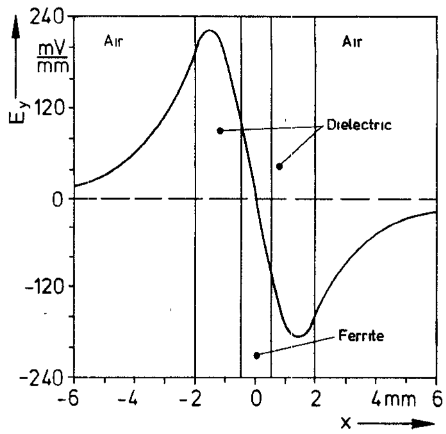

Fig. 3. The x dependence of the y component of the electric field strength of mode I for the guide shown in Fig. 3.

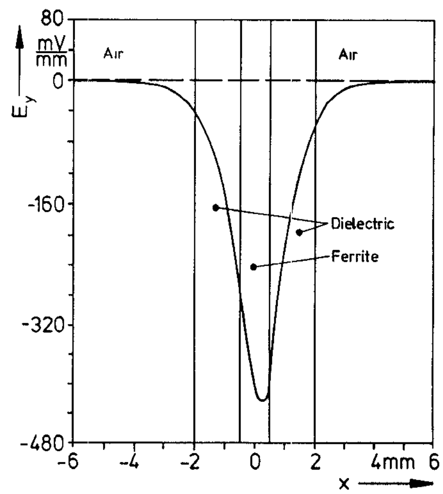

Fig. 4. The x dependence of the y component of the electric field strength of mode II for the guide shown in Fig. 3.

Because Q must be real, the following must be valid:

$$\{P_{\mathrm{A}}^{\mathrm{I,II}}\}^2 \geqslant 4P_{\mathrm{B}}^{\mathrm{I,I}}P_{\mathrm{B}}^{\mathrm{II,II}}. \tag{14}$$

If (14) is not true, the problem cannot be described exactly by this method, but it can be extended to solve such problems in a similar way.

In most cases the field distribution in each part of the asymmetrical structure is more similar to that of a single line for mode II (quasi-odd mode) than for mode I (quasi-even mode, see Fig. 4). Consequently, the upper sign in (13) is used. The power division factor k can be defined as

$$k = \left(P_{\mathrm{B}}^{\mathrm{I,I}} + Q^2 P_{\mathrm{B}}^{\mathrm{II,II}}\right)\Big/\left(P_{\mathrm{A}}^{\mathrm{I,I}} + Q^2 P_{\mathrm{A}}^{\mathrm{II,II}}\right)$$
$$= Q P_{\mathrm{A}}^{\mathrm{I,II}}\Big/\left(P_{\mathrm{A}}^{\mathrm{I,I}} + Q^2 P_{\mathrm{A}}^{\mathrm{II,II}}\right). \tag{15}$$

This factor describes the relation between the magnitude of the z-dependent part of the transported power and its constant part on guide A (see (7)). This is equal to the relation between the constant parts on guide B and guide A. Thus (7) and (8) can be written

$$P_{\mathrm{A}}(z)/P_0 = \{1 + k\cos(\pi z/L)\}/\{1+k\} \tag{16}$$

$$P_{\mathrm{B}}(z)/P_0 = \{k - k\cos(\pi z/L)\}/\{1+k\}. \tag{17}$$

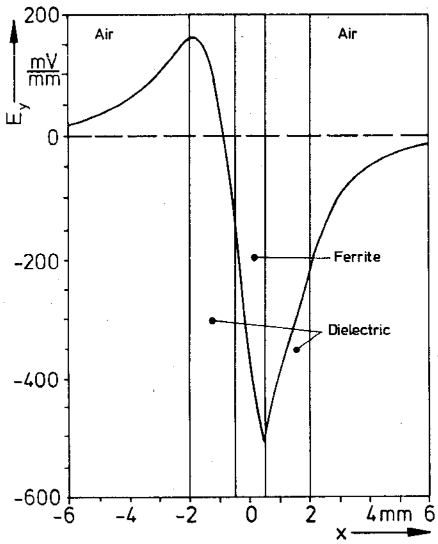

Fig. 5. The x dependence of the y component of the electric field strength of the superposition of the two modes from Figs. 3 and 4 at $z = 0$.

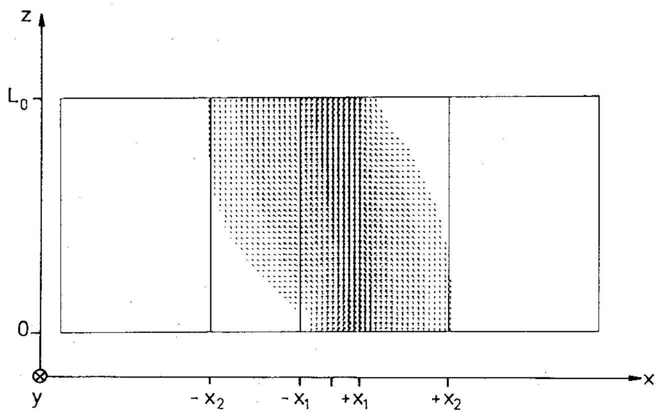

Fig. 6. Longitudinal power flow (Poynting vector) of the superposition of the two fundamental modes.

III. Principal Results

From (15) the power division factor k may become larger than one, in which case the power transported on guide A is negative. This means that the power on this transmission line is transported in the $-z$ direction. Nevertheless, inspecting both guides together the power transport still goes in the $+z$ direction. If k is less than or equal to one, the magnitudes of the scattering parameters at $z = l$ can be determined:

$$|S_{21}| = \sqrt{P_A(z=l)/P_0} = \sqrt{\{1 + k\cos(\pi l/L)\}/\{1+k\}} \quad (18)$$

$$|S_{31}| = \sqrt{P_B(z=l)/P_0} = \sqrt{\{k - k\cos(\pi l/L)\}/\{1+k\}}. \quad (19)$$

Fig. 2 shows a diagram of the scattering parameters versus the normalized coupler length for a power division factor $k = 0.8$. Because the magnitude of S_{21} never becomes equal to zero, it can be recognized that an incomplete periodical coupling is submitted.

The phases of the S parameters can be calculated from the electromagnetic fields ((4) and (5)). It has been assumed that they have zero phases for $z = 0$. Therefore the phases of these fields at $z = l$ and at significant coordinates (x, y) of each line are equal to the phases of the corresponding scattering parameters.

If the structure is electromagnetically symmetric, $P_A^{I,I}$ and $P_B^{I,I}$, as well as $P_A^{II,II}$ and $P_B^{II,II}$, are identical. This implies that k becomes one. Thus (18) and (19) of this generalized theory converge into (1) and (2) of the common coupler theory.

The x dependences for the E_y field components of the first two single modes on the guide are shown in Figs. 3 and 4. The superposition of these two modes described above is shown in Fig. 5. The resulting power flow in the z direction is shown in Fig. 6. At $z = 0$ no part of the power is transported between $-x_2 \leqslant x \leqslant -x_1$. For increasing values of z, part of the power flow changes to this line, but at $z = L_0$ a nonnegligible part of the power is still transported between $+x_1 \leqslant x \leqslant +x_2$. This demonstrates the incomplete coupling at electromagnetically asymmetrical coupled lines.

IV. Conclusions

The method presented here is an extension of the well-known even- and odd-mode analysis for symmetrical coupled lines. The power consideration allows an integral formulation of the problem and can also be used for asymmetrical problems. In these cases the coupling is incomplete in the sense that only a part of the total power is coupled between the two guides. The principles of such coupler applications have been demonstrated using the example of an image guide coupler with a premagnetized ferrite slab.

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Improved Theory for E-Plane Symmetrical Tee Junctions

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***Abstract*—We examine Lewin's theory, which describes an E-plane symmetrical tee junction by a peculiar equivalent circuit with only three parameters. It is shown that although his theory is formally correct, its circuit parameters depend on the amplitudes of reflected waves. An improved theory corrects this fault.**

Manuscript received November 11, 1987; revised October 10, 1988.
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IEEE Log Number 8825373.

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